Variational Inequalities, Nash Equilibrium Problems and Applications

Catania, July 11-12, 2024

ABSTRACT BOOKLET

VINEPA 2024

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Università degli Studi di Catania

VINEPA 2024 — Book of Abstracts

Variational Inequalities, Nash Equilibrium Problems and Applications

The workshop aims at presenting the state-of-the-art and current research on Variational Inequalities, Nash Equilibrium Problems and Applications.

Recently, there has been a sharp increase in interest in variational inequalities. They are now one of the most challenging and dynamic topics of mathematics, and represent excellent tools in the study of real-world problems. In fact, they cover a large variety of applications of extreme importance related to engineering, computer science, mathematical physics, statistics, economics, financial networks and generalized complementarity problems.

In addition, Nash equilibrium problems, naturally associated with variational inequalities, experienced a surprising development. This leads to finding effective solutions to until now unsolved problems in numerous real-life situations, such as oligopoly models, environmental problems, network problems and infrastructure problems.

Variational methods and game theory are also used in resource orchestration in a 5G/6G environment for creating and managing network slices, NVFs or services; in network optimization problems, with applications to logistics and transportation, to improve the eco-sustainability of last mile logistics using UAVs; in computer vision, machine learning, pattern recognition and artificial intelligence.

The aim of the workshop is to bring together scholars working on both theoretical and computational issues, present results having the potential of solving concrete problems, and thus try to fill the gap between theory and practice.

This booklet contains the abstracts of the communications presented during VINEPA 2024.

Catania, July 2024

Gabriella Colajanni
Patrizia Daniele
Sofia Giuffrè
Fabio Raciti
Laura R.M. Scrimali

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INVITED COMMUNICATIONS

The limit dynamic of the Su-Boyd-Candès accelerated gradient system when the asymptotic vanishing damping coefficient α becomes large: a singular perturbation approach

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Abstract
In a real Hilbert space setting, we concentrate on the continuous dynamical system introduced by Su, Boyd, and Candès as a low-resolution ordinary differential equation (ODE) version of Nesterov's accelerated gradient method (NAG). This inertial system, represented as AVD_α, is driven by the gradient of the function f that is subject to minimization and features a damping mechanism with an asymptotic vanishing coefficient of the form α/t, where $\alpha \geq 3$. Selecting a sufficiently large α is pivotal for ensuring the desirable asymptotic convergence characteristics of the trajectories. Specifically, for a general convex function f, choosing $\alpha > 3$ ensures an asymptotic convergence rate of the values at $o(1/t^2)$, in addition to the convergence of the trajectories towards the optimal solutions. In the case of strongly convex functions f, the convergence rate asymptotically achieves the order of $1/t^{\frac{2\alpha}{3}}$, improving with increasing α. To elucidate the influence of the parameter α on the convergence properties of AVD_α, our analysis reveals that an appropriate time scaling of AVD_α yields trajectories that closely resemble those produced by the continuous steepest descent method associated with f, particularly when α is substantially large. This approach highlights a singular perturbation phenomenon as the analysis transitions from a second-order evolution equation to a first-order one. Such a transition is instrumental in comprehending the shift in the convergence rate from $1/t$ to $1/t^2$, distinguishing the steepest descent method from NAG.

Revisiting Variational Inequalities, Nash Equilibrium Problems and their Applications thanks to local reproducibility concept

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Abstract
It is well known that (quasi-)variational inequalities and Nash equilibrium problems play a fundamental role in mathematics and in particular for applications in economics and engineering.
Our aim here is to revisite a part of the theory developed for those problems through the lens of local solution concepts. This will be done thanks to the new concept of local reproducibility for set-valued maps.

Agricultural Supply Chain Networks: From International Trade to Resilience

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Abstract
Agricultural supply chain networks are essential to food security and are a major component of global trade. In this talk, I will describe how optimization and game theory, along with variational inequalities, are being utilized to model and solve agricultural supply chain problems subject to a spectrum of trade instruments; how to capture fresh produce quality in multitiered supply chains, and how to measure resilience in order to mitigate against disruptions under conditions such as the pandemic, climate change, and wars. I will also discuss how we can better promote our research contributions in order to influence policy and effect positive change.

Optimality conditions in optimization under uncertainty

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Abstract
Most optimization problems involve uncertain data due to measurement errors, unknown future developments and modelling approximations. Stochastic optimization assumes that the uncertain parameter is probabilistic. An other approach is called robust optimization which expects the uncertain parameter to belong to a set that is known prior. In this talk, we consider scalar optimization problems under uncertainty with infinite scenario sets. We apply methods from vector optimization in general spaces, set-valued optimization and scalarization techniques (see [1]) to derive necessary optimality conditions for solutions of robust counterpart problems (compare [2]).

Keywords: Robust Optimization, Nonlinear Scalarization, Vector Optimization.

References:
[1] K. Klamroth, E. Köbis, A. Schöbel, Chr. Tammer, *A unified approach to uncertain optimization*, European J. Oper. Res. 260, pp. 403-420, 2017.
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Fixed points of regular set-valued mappings in quasi-metric spaces

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Abstract
This talk focuses on the existence of fixed points for set-valued mappings defined on quasi-metric spaces complete or non-complete. Quasi-metrics play a central role in the theory of spaces of homogeneous type and have numerous recent applications both in pure and applied mathematics, for example, in the questions of existence and uniqueness of Hamiltonacobi equations, in rate-independent models for plasticity, shape-memory alloys, models for material failure, automated taxonomy construction. In the context mentioned above, we use the notion of orbital regularity/ orbital pseudo-Lipschitzness of set-valued mappings defined by Ioffe to obtain the estimate of the distance from a given point to the set of fixed points of the mapping under consideration. Existence of double fixed points, coincidence points of pairs of set- valued mappings from a quasi-metric space into another one will also be given. Moreover, the existence of cyclic fixed points for a collection of set-valued mappings and estimations of the distance from a given point to the set of fixed points or the distance between two sets of fixed points will be also provided. Our results generalize recent results given by Ait Mansour et al. [1], Dontchev & Rockafellar [4], Ioffe [2], Lim [3] and Nguyen Huu Tron [5].

Keywords: Regularity, Pseudo-Lipschitzness contraction, orbital regularity, coincidence point, fixed point, cyclic fixed point, quasi-metric space.

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CONTRIBUTED COMMUNICATIONS

A PMP-based method for solving dynamical Nash games

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Abstract
A novel sequential quadratic Hamiltonian (SQH) scheme designed for tackling open-loop non zero-sum differential Nash games is presented. This method, rooted in Pontryagin's maximum principle, extends the successive approximations strategy effectively within the domain of Nash games. In the SQH method, the Hamilton Pontryagin functions incorporate a quadratic penalty term, and the Nikaido-Isoda function serves as a selection criterion. The central concept of the SQH scheme lies in the finite-dimensional Nash game derived from the PMP characterization of Nash games, which holds true for any fixed time. We identify a class of problems where this finite-dimensional game has a unique solution and provide theoretical results establishing the well-posedness of our proposed scheme for this class of games. Additionally, since solving coupled Riccati equations readily yields solutions for unconstrained LQ Nash games, they serve as a convenient benchmark for our method.
Furthermore, we demonstrate the applicability of our approach to linear-quadratic Nash games and their variants, including those with control constraints, weighted L^1 costs for players' actions, and tracking objectives. These applications corroborate the theoretical statements.

***Keywords*:** Nash games, Pontryagin maximum principle, numerical methods.

References
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Lawful Interceptions through Data Transmission Optimization

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Abstract
In today's world, Internet of Things (IoT) devices are pervasive in our daily lives. They are constantly connected to the Internet, present in homes, workplaces, and used as personal accessories. The problem of privacy arises from the vast amounts of personal and sensitive data that IoT devices collect about their users, which are transmitted to remote servers [1,2]. The use of IoT Safeguards, often integrated into the home router as a commercial device, has been proposed to protect users' privacy by monitoring Internet traffic generated from IoT devices [3]. In this context, we deal with a new aspect that regards the use of IoT devices for lawful interceptions of a person in her/his home, where an IoT Safeguard is used for protection. We analyze how the data intercepted inside the home (such as audio and/or video recorded in the intercepted environment) can be transmitted to the entity conducting the interception, which is placed outside the home, bypassing the protections provided by an IoT Safeguard. The problem we intend to solve is to maximize the throughput of the intercepted data overcoming the IoT Safeguard protection and minimizing the probability that the IoT Safeguard, by inspecting the network traffic, can detect the information leakage. For this purpose, we intend to analyze the impact of the following parameters: the number and size of the network packets, the amount of data to be transmitted, the network latency, the packet loss rate, and the packet error rate.

Keywords*:* IoT, privacy, information leakage, network packet, latency.

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A new proof for the existence of a Nash equilibrium

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Abstract
We present a new proof for the existence of Nash equilibria that needs no fixed point theorem. Instead we introduce the notion of distributed equilibria, which generalizes Nash equilibria, and show that arbitrarily small distributed equilibria always exist. By means of compactness, we then show that Nash equilibria always exist.

Keywords: Nash equilibrium, Noncooperative Game.

References:
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Affine relaxations of the best response algorithm in ratio-bounded games: approximation of Nash and bilevel Nash equilibria

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Abstract
Best response dynamics is one of the most explored iterative scheme to find Nash equilibria. It consists in updating the strategy of player i by selecting a minimizer of player i's payoff function given the strategy of the other players that come from the previous step. In this presentation we deal with the affine relaxations of the best response algorithm in games with two players whose strategy sets are real Hilbert spaces. In order to investigate the convergence of any type of affine relaxation, we consider a new class of convex games, called ratio-bounded games [4]. This class contains games broadly used in literature, like weighted potential games and zero-sum games, both in finite and in infinite dimensional setting. Its definition relies on a unifying property and on three associate key-parameters explicitly related to the data. Depending on how the parameters are ordered, we provide a classification of the ratio-bounded games in four subclasses such that, for each of them, the following issues are answered: existence and uniqueness of the Nash equilibria, global convergence of the affine relaxations of the best response algorithm, estimation of the related errors and identification of the algorithm with the highest speed of convergence. Then, we present a numerical scheme based on a direct-search method to approach both Nash equilibria of ratio-bounded

games and bilevel Nash equilibria when ratio-bounded games are involved in the lower level.

***Keywords*:** approximation of Nash equilibria, affine relaxation of the best response algorithm, bilevel Nash equilibrium problem.

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On Contracting Dynamics for Convex Optimization Problems

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Abstract
Optimization problems (OPs) are fundamental in science and engineering, with a wide range of applications spanning from control theory to game theory. A rising approach for addressing possibly timevarying OPs is to synthesize continuous-time dynamical systems, akin to recurrent neural networks, that converge to equilibria that are also optimal solutions. Consequently, significant research effort has been made to characterize the stability and convergence rates of these systems, along with their robustness against uncertainty. Remarkably, these desirable properties such as convergence for time-invariant (tracking for time-varying) dynamics and robustness to noise, can be established by ensuring that the system is contracting. Contracting dynamics are robustly stable, computationally-friendly, and enjoy many other useful properties.
In this context, we focus on solving convex optimization problems with a unique minimizer via continuous-time dynamical systems. We show that these OPs lead to dynamics that are contracting. For such dynamics, we characterize their convergence behavior. Specifically, we show that convergence is (i) exponential for OPs with strongly convex costs and (ii) linear-exponential for OPs with only convex costs. Next, we provide input-to-state stability conditions for the dynamics and illustrate the effectiveness of our results via several applications. We conclude by exploring the advantages of using contracting theory for solving variational inequalities.

Keywords: Contraction Theory, Convex Optimization, Stability, Nonlinear Dynamical Systems, Recurrent Neural Networks.

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URL: https://fbullo.github.io/ctds

On existence of general equilibrium with transfers and other-regarding preferences: variational inequality approach

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Abstract
In this talk, we present a pure exchange economy where households can engage in voluntary commodity transfers with each other. Our general equilibrium model considers the interdependency of households' preferences on their own consumption and the wealth of other households. We establish the existence of solutions for a non-convex model (where households' preferences are not assumed to be convex) by proposing a reformulation of the model in terms of a quasi-variational inequality. We achieve the existence of equilibrium by assuming an upper bound on the transfers and without imposing compactness constraints on the consumption set.

Keywords: general equilibrium and altruism, variational inequality problem, maximal elements.

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A game-theoretical approach to a financial equilibrium model with uncertain parameters

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Abstract
We propose a game-theoretical formulation of a classical financial equilibrium model involving *m* economic sectors and *n* financial instruments [1]. The new approach provides an explanation of the mechanism of the (equilibrium) price formation. Our game falls in the class of potential games and its *price of anarchy* is equal to 1. We then refine the model by considering the possibility that some parameters are not deterministic but only their probability distributions are known and formulate our problem as a random variation inequality [2].

Keywords **:** financial equilibrium, variation inequality, game theory.

References :
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[2] Joachim Gwinner, Baasansuren Jadamba, Akhtar A. Khan, Fabio Raciti, *Uncertainty Quantification in Variational Inequalities Theory, Numerics, and Applications,* CRC Press, 2022.

Ekeland variational principle and its equivalents in premetric spaces

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Abstract
We consider a generalized metric (premetric) function without symmetry property and without triangle inequality. We formulate and prove Long Orbit or Empty Value (LOEV) principle in topological spaces with premetric function. We establish that LOEV principle is equivalent to the Sigma-semicompleteness of the space. Variants of Ekeland variational principle, Caristi-Kirk fixed point theorem and Takahashi variational principle in Sigma-semicomplete premetric spaces are proved and their equivalence to the LOEV principle is established.

Keywords: Multi-valued mapping, Fixed point, Variational principle, Sigma-semicompleteness.

References:
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A Variational Approach to Trust and Reputation Systems

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Abstract
Trust and reputation systems are crucial in contemporary decentralised environments [1], fostering collaboration and risk management across various online interactions [2]. Our research aims to introduce a variational formulation approach for modeling and analyzing these systems. By framing trust and reputation as variational problems, our approach offers a fresh perspective on understanding the fundamental mechanisms governing trust establishment. The variational formulation provides a mathematical framework to determine equilibrium weighted trust values, considering that each trustee seeks to maximize its utility while minimizing costs. Through a plethora of simulations, we demonstrate the effectiveness of this variational formulation in modeling trust and reputation systems. Insights derived from our approach provide valuable guidance for designing and implementing more robust and efficient trust and reputation mechanisms in decentralized environments.

***Keywords*:** weighted trust, equilibrium conditions, variational formulation.

References:

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A preference equilibrium problem

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Abstract
In this talk we consider a production and consumption market, where we suppose the possible presence of organized crime infiltration in the firms; consequently, the presence of the mafia affects the consumption and the production of the market. Then, we propose a theoretical model of general equilibrium in the presence of organized crime in the firms.
The problem fits in a general economic equilibrium problem where the equilibrium conditions are given by the agents' preference allocations and suitable market clearing conditions. To study the problem, we rewrite it as a particular preference equilibrium problem; hence we analyze it by using a suitable quasi-variational inequality formulation*

Keywords: organized crime infiltration, preference equilibrium problem.

References:
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Random optimal control equilibrium problem via inverse stochastic variational inequalities

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Abstract
The main objective of the research is to analyze how policymakers influence the random oligopolistic market equilibrium problem. To this purpose, random optimal control equilibrium conditions are introduced. Since the random optimal regulatory tax is characterized by an inverse stochastic variational inequality, existence and well-posedness results on such an inequality are proved. At last a numerical example is discussed.

Keywords: Random optimal control equilibrium problem, Inverse stochastic variational inequalities, Existence results, Well-posedness analysis.

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Seeking an algorithm to projected solution of quasi variational inequality problems with non-self constraint map

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Abstract
In a Hilbert space, we present an algorithm that leads to the projected solutions of quasi variational inequality problems while the operator deals with the problem is Lipschitz continuous and strongly monotone. The algorithm is based on an interaction between resolvent operator and reflected resolvent operator.

Keywords: projected solutions, quasi variational inequality, non-self constraint map.

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Multi-layer 5G Network Slicing with UAVs: an optimization model

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Abstract
In this paper, we provide a closed-loop supply chain network model for the provision of 5G services. It is based on a three-tiered closed loop network in which, in the forward chain, the users or de-vices on the ground require services, executed by a fleet of UAVs orga-nized as a FANET, controlled by a fleet of manager UAVs, whose role is to receive service requests and send them to executor UAVs. In the reverse chain, the fleet of executor UAVs, once the requests are performed, provide the services to devices and users on the ground. Through a system optimization perspective, service providers seek to maximize their profit, given by the difference between the revenue associated with the provision of services and the costs due to the transmission and the execution of services.
Considering the trustworthiness levels of each UAV belonging to the fleet at the highest level of the network, service providers also seek to minimize the investment costs to increase such levels and the penalty associated with not executed services due to unreliability of the executor UAVs. For the proposed optimization problem, we provide a variational formulation and some numerical simulations are performed.

Keywords: Closed-Loop Supply Chain Network, Trustworthiness of 5G Networks, UAVs, Variational Inequality

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A variational approach to preference equilibrium problems under weak continuity assumptions in Banach spaces

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Abstract
Optimization and equilibrium problems have been extensively studied when the inovlved preference relations admit a representation by means of real-valued functions. Although these problems have been analyzed under very minimal assumptions on the representation function, this context could appear to be quite restrictive in some practical situations. By using tools of variational analysis and by weakening some continuity properties, we aim to provide, in an infinite dimensional setting, an alternative approach to deal with the study of preference relations without (necessarily) representation means of real-valued functions. As an application of our theoretical developments, we analyze a particular preference equilibrium problem (of which a preference maximization problem is a particular case) by using a suitable quasi-variational inequality formulation.

Keywords: Preference Relation, Normal Cone Operator, Quasi-variational inequality.

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A MILP formulations and a branch-and-cut approach for several truck-and-drone logistics problem

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Abstract
Nowadays, truck-and-drone problems represent one of the most studied classes of vehicle routing problems. The Flying Sidekick Traveling Salesman Problem (FS-TSP) is the first optimization problem defined in this class [4]. Since its definition, several variants have been proposed differing for the side constraints related to the operating conditions and for the structure of the hybrid truck-and-drone delivery system. Determining the optimal solution of most of these routing problems is a very challenging task, due to the vehicle synchronization issue and drone hovering [2].
On this basis, this work provides a new arc-based integer linear programming formulation for the FS-TSP [1]. The proposed formulation can be easily extended to deal with two interesting variants of the FS-TSP, namely: the Traveling Salesman Problem with Drone and Lockers, where customers are served either by truck or a drone directly at their houses or use self-pickup facility; the Truck-Drone Team Logistics Problem, where a drone can serve multiple customers during in each flight.

The solution of the proposed formulation and its variants required the development of a branch-and-cut solution approach based on new families of valid inequalities and variable fixing strategies.
We tested the proposed approach on different sets of benchmark instances. The experimentation shows that the proposed method is competitive or outperforms the state-of-the-art approaches [3].

Keywords**:** truck-and-drone; branch-and-cut; MILP formulation; last-mile logistics.

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On Cournot oligopolies with biconcave price functions

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Abstract
It has turned out, that when dealing with Cournot equilibria, generalized concavity is quite useful.
In this context Ewerhart's uniqueness theorem [1] is a milestone. This theorem provides sufficient conditions for equilibrium uniqueness in case of biconcave (essential) price functions. It presupposes that industry revenue is continuous which excludes for example a unit-elastic price function. By referring to recent results for Nash equilibria of sum-aggregative games [2,3] we derive a variant of Ewerhart's uniqueness theorem that allows for such an discontinuity.

Keywords **:** Cournot oligopoly, biconcavity, equilibrium uniqueness.

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